The Origins of Sedentism

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Appendix

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Proof of Proposition 1.

In all cases the solution in (2) is unique due to strict concavity, and first order conditions (FOC) for a maximum are also sufficient.

(a) By A1 there is a unique $x_f > 0$ satisfying the condition in part (a) of Proposition 1.

(i) We have $f'(L) \geq f'(x_f) \equiv kg'(0)$. Thus $L_f = L$ and $L_g = 0$ satisfies the FOC for a maximum.

(ii) The solution cannot have $L_f = 0$ because then the FOC implies $kg'(L) \geq f'(0) > kg'(0) > kg'(L)$, which is a contradiction. The solution cannot have $L_g = 0$ because then the FOC implies $f'(L) \geq kg'(0) \equiv f'(x_f) > f'(L)$, which is a contradiction. It follows that $L_f > 0$ and $L_g > 0$. The FOC for an interior solution is $f'(L_f) = kg'(L_g)$.

(b) The proof parallels (i) and (ii) in part (a).

(c) The proof parallels (ii) in part (a).

(d) H(L, k) is continuous in (L, k) by the theorem of the maximum. It is increasing in L for a fixed k because f is increasing in L_f and g is increasing in L_g. To show that H is strictly concave in L, fix $k > 0$, choose any $L' \neq L''$, and choose any $\mu \in (0, 1)$. Let (L_f', L_g') be optimal for the total labor supply L' and let (L_f'', L_g'') be optimal for the total labor supply L''. Define $L_f^* \equiv \mu L_f' + (1-\mu)L_f''$ and $L_g^* \equiv \mu L_g'$

+ (1-μ)L_g''. Notice that (L_f*, L_g*) is a feasible allocation of the total labor supply L* = μL′ + (1-μ)L″. This implies H(L*, k) ≥ f(L_f*) + kg(L_g*) > μf(L_f') + (1-μ)f(L_f'') + μkg(L_g') + (1-μ)kg(L_g'') = μH(L′, k) + (1-μ)H(L″, k). The strict inequality in this sequence follows because f(L_f*) > μf(L_f') + (1-μ)f(L_f'') due to the strict concavity of f, and kg(L_g*) > μkg(L_g') + (1-μ)kg(L_g'') due to the strict concavity of g. This establishes that H(L, k) is strictly concave in L.

(e) Due to H(0, k) = 0, the strict concavity of H implies H(μL, k) > μH(L, k) for all L > 0 and μ ∈ (0, 1). This yields H(μL, k)/μL > H(L, k)/L for all L > 0 and μ ∈ (0, 1). Thus h(L, k) ≡ H(L, k)/L is decreasing in L.

(f) (i) Let [L_f(L), L_g(L)] be the optimal allocation for L > 0. One and only one of parts (a), (b), or (c) above must apply. Suppose (a) applies. Then for sufficiently small L > 0 we have H(L, k) = f(L) and h(L, k) = f(L)/L. This implies that as L → 0, we have h(L, k) → f′(0) > kg′(0). Therefore h(0, k) = max {f′(0), kg′(0)}. The proofs for (b) and (c) are similar.

(ii) For a fixed k > 0, whenever L is sufficiently large we have L_f(L) > 0 and L_g(L) > 0 with f′[L_f(L)] = kg′[L_g(L)]. From part (e) above, h(L, k) is decreasing in L. Suppose there is a lower bound δ > 0 such that h(L, k) ≥ δ for all L > 0. This implies f[L_f(L)]/L + kg[L_g(L)]/L ≥ δ > 0 for all L > 0. We have f(L_f)/L_f → 0 as L_f → ∞. This is obvious if f has a finite upper bound. If f is unbounded, then using f′(L_f) → 0 as L_f → ∞ from A1 gives the same result. Likewise g(L_g)/L_g → 0 as L_g → ∞. The lower bound δ > 0 implies that L_f(L) → ∞ and L_g(L) → ∞ cannot both hold when L → ∞. Therefore one or the other must have a finite upper bound M > 0. Suppose L_f(L) ≤ M for all L. Then f′[L_f(L)] ≥ f′(M) > 0 for all L > 0. Since

$L_f(L)$ has a finite upper bound, $L_g(L) \to \infty$ must hold as $L \to \infty$. Thus $kg'[L_g(L)] \to 0$ as $L \to \infty$. For sufficiently large L, this contradicts the first order condition $f'[L_f(L)] = kg'[L_g(L)]$. The same is true if $L_g(L)$ has a finite upper bound. Thus neither $L_f(L)$ nor $L_g(L)$ has an upper bound, so there is no lower bound $\delta > 0$ and $h(L, k) \to 0$ as $L \to \infty$.

Proof of Proposition 2.

(a) For all $y \geq \theta_A h(0)$ we have $\eta(y/\theta_A) = 0$ due to (3a). Also, $\theta_A > \theta_B$ implies $y \geq \theta_B h(0)$ so we have $\eta(y/\theta_B) = 0$ due to (3b). Therefore $D(y) = 0 < N$ and no such y solves (4). For $y \leq \theta_A h(0)$, D(y) is continuous and decreasing because $\eta(y/\theta_A)$ is continuous and decreasing while $\eta(y/\theta_B)$ is continuous and non-increasing. Also, $D(y) \to \infty$ as $y \to 0$ because $h(L) \to 0$ as $L \to \infty$ from Proposition 1(f). Because $N > 0$ is finite, $D[\theta_A h(0)] = 0$, $D(0) = \infty$, and D(y) is continuous and decreasing on $(0, \theta_A h(0)]$, there is a unique $y(N) \in (0, \theta_A h(0))$ such that $D[y(N)] = N$.

(b) Consider the unique value of y from part (a) that solves (3c) and choose L_A and L_B as in Proposition 2(b). The fact that y solves (3c) implies that condition (c) in the definition of SRE holds. Using $y < \theta_A h(0)$ when $D(y) = N$ as in Proposition 2(a), along with $L_A = \eta(y/\theta_A) > 0$ as in (3a), implies $\theta_A h(L_A) = y$ so that condition (a) in the definition of SRE holds. From (3b), either (i) $L_B = \eta(y/\theta_B) > 0$, which implies $\theta_B h(L_B) = y$, or (ii) $L_B = \eta(y/\theta_B) = 0$, which implies $\theta_B h(0) \leq y$. In either situation, condition (b) in the definition of SRE holds. This shows that (y, L_A, L_B) is a SRE. To show that it is unique, suppose (y', L_A', L_B') is a different SRE. Condition (a) in the definition of SRE implies $L_A' = \eta(y'/\theta_A) > 0$ and condition (b) in the definition of SRE implies either (i) $L_B' = \eta(y'/\theta_B) > 0$ or (ii) $L_B' = \eta(y'/\theta_B) = 0$ with $y' \geq \theta_B h(0)$. Condition (c) in the definition of SRE implies $\lambda L_A' + (1-\lambda)L_B' = N$. This implies $D(y') \equiv \lambda\eta(y'/\theta_A) + (1-\lambda)\eta(y'/\theta_B) = N$. But from Proposition 2(a) there is a unique solution to (3c), so it must be true that $y = y'$. Hence $L_A' \neq L_A$ or $L_B' \neq L_B$ or both. However, this is impossible because for a given value of y there is a unique solution for L_A from (a) in the definition of SRE and the same is true for L_B from (b) in the definition of SRE.

(c) Continuity of y(N) follows from the continuity of D(y) in (3c). y(N) is decreasing because D(y) is decreasing over the relevant range. To show that $y(N) \to 0$ as $N \to \infty$, use (3c) to write the identity $D[y(N)] \equiv \lambda\eta[y(N)/\theta_A] + (1-\lambda)\eta[y(N)/\theta_B] \equiv N$ where y(N) is decreasing. Suppose there is a lower bound $\delta > 0$ such that $y(N) \geq \delta$ for all $N > 0$. Because η is decreasing, $D[y(N)] \leq \lambda\eta(\delta/\theta_A) + (1-\lambda)\eta(\delta/\theta_B)$ for all $N > 0$. Choosing any N that exceeds the right hand side of this inequality gives a contradiction. Thus there is no such lower bound and $y(N) \to 0$ as $N \to \infty$.

Proof of Proposition 3.

(a) Suppose $\theta_A h(0, k) > y^*$. By Proposition 1 there is an $L_A > 0$ such that $\theta_A h(L_A, k) = y^*$ or equivalently $L_A = \eta(y^*/\theta_A, k) > 0$. Set $L_B = \eta(y^*/\theta_B, k) \geq 0$ and $N = \lambda L_A + (1-\lambda)L_B > 0$. The triple (L_A, L_B, N) is an LRE because (L_A, L_B, y^*) is a SRE for N. Any other LRE must have the same (L_A, L_B) in order to satisfy conditions (a) and (b) in the definition of SRE at the income y^*. It must therefore have the same N to satisfy condition (c) in the definition of SRE. This establishes uniqueness.

(b) Suppose there is some (L_A, L_B, N) with $N > 0$ that is an LRE. From condition (a) in the definition of SRE we must have $\theta_A h(L_A, k) = y^*$ with $L_A > 0$. However, this implies $\theta_A h(0, k) > y^*$, which contradicts the assumption $\theta_A h(0, k) \leq y^*$.

Proof of Proposition 4.

(a) Necessity. From (b) in the definition of VLRE, a necessary condition for a VLRE with such a value of k is $L_{Ag} = L_{Bg} = 0$. Using Proposition 1, $L_{Ag} = 0$ occurs if and only if $f'(L_A) \geq kg'(0)$. From (a) in the definition of VLRE, another necessary condition is that (L_A, L_B, N) form an LRE for the given k. In turn, this requires that (L_A, L_B, y^*) form a SRE for $N > 0$. From (a) in the definition of SRE, this implies $\theta_A h(L_A, k) = y^*$. Due to $L_{Ag} = 0$ this reduces to $\theta_A f(L_A)/L_A = y^*$. There is an $L_A > 0$ satisfying this equation iff $\theta_A f'(0) > y^*$. Together these results show the necessity of the conditions stated in Proposition 4(a).

Sufficiency. Suppose the conditions in Proposition 4(a) are satisfied. Compute L_A, L_B, and N as in the proposition. We need to show that this gives a non-null VLRE. $L_A > 0$ implies $N > 0$ so any VLRE will be non-null. Condition (b) in the definition of VLRE is satisfied because (i) $f'(L_A) \geq kg'(0)$ implies $L_{Ag} = 0$ from Proposition 1; and (ii) $\theta_B < \theta_A$ implies $L_B < L_A$, which implies $f'(L_B) \geq kg'(0)$, and this in turn implies $L_{Bg} = 0$ from Proposition 1. Condition (a) in the definition of VLRE is satisfied because the definition of LRE is satisfied.

(b) When $k = k^*$, condition (b) in the definition of VLRE is satisfied. Condition (a) in the definition of VLRE reduces to the conditions for a (non-null) LRE. From Proposition 3(a), these conditions can be satisfied iff $\theta_A h(0, k^*) > y^*$. When this inequality holds, condition (a) in the definition of SRE implies that $L_A > 0$

satisfies $\theta_A h(L_A, k^*) = y^*$. Condition (b) in the definition of SRE implies that if $\theta_B h(0, k^*) > y^*$ then $L_B > 0$ satisfies $\theta_B h(L_B, k^*) = y^*$; otherwise $L_B = 0$. Finally, condition (c) in the definition of SRE gives $N = \lambda L_A + (1-\lambda)L_B > 0$.

Proof of Proposition 5.

By Proposition 4(a), if $k < k^*$ there is a non-null VLRE iff $\theta_A f'(0) > y^*$ and the value of L_A such that $\theta_A f(L_A)/L_A = y^*$ gives $f'(L_A) \geq kg'(0)$. This gives conditions (9a) and (9c). It is automatic that only hunting is active. Adding the requirement that only sites of type A are used implies $L_B = 0$. By Proposition 4(a) this holds iff $\theta_B f'(0) \leq y^*$. This gives condition (9b).

Proof of Proposition 6.

(a) In the baseline equilibrium $L_B^0 = 0$ because type-B sites are not used. Thus $N^0 = \lambda L_A^0 > 0$ where L_A^0 is the baseline population at a type-A site. We have $f'(L_A^0) \geq k^0 g'(0)$ from (9c). Because $N^0 = \lambda L_A + (1-\lambda)L_B$ is fixed in the short run and $L_B \geq 0$ under the new climate regime, we must have $L_A \leq L_A^0$ under the new climate regime. Because there is no change in k^0, we have $f'(L_A) \geq k^0 g'(0)$ so gathering cannot be used at the type-A sites. Because $L_B < L_A$ in every SRE, we have $f'(L_B) > k^0 g'(0)$ so gathering cannot be used at type-B sites either.

(b) In the baseline equilibrium regional population is $N^0 = \lambda L_A^0 = \lambda\eta(y^*/\theta_A^0, k^0)$ where the second equality follows from condition (a) in the definition of SRE and (3a). From (5), type-B sites are used in period $t = 0$ under the new climate regime iff $N^0 > N^*(k^0) \equiv \lambda\eta[\theta_B^* h(0, k^0)/\theta_A^*, k^0]$ or equivalently $\eta(y^*/\theta_A^0, k^0) > \eta[\theta_B^* h(0, k^0)/\theta_A^*, k^0]$. Because $\eta = h^{-1}$ is decreasing in its first argument for a fixed k^0, this holds iff $y^*/\theta_A^0 <$

$\theta_B{}^*h(0, k^0)/\theta_A{}^*$. Due to (9c) for baseline equilibrium, $h(0, k^0) \equiv \max\{f'(0), k^0g'(0)\} = f'(0)$. Substituting this into the previous inequality gives the result in Proposition 6(b).

Proof of Proposition 7.

First we prove some preliminary results. Define $L_h > 0$ to satisfy $\theta_A{}^*h(L_h, k^0) \equiv \theta_B{}^*h(0, k^0)$. This L_h exists and is unique due to $\theta_A{}^* > \theta_B{}^*$ and Proposition 1(d)-(f). By the definition of SRE we have $L_B = 0$ when $0 \leq L_A \leq L_h$ and $L_B > 0$ when $L_A > L_h$.

Observe from (9c) in Proposition 5 that $f'(0) > k^0g'(0)$ holds. Therefore $h(0, k^0) = f'(0)$ from Proposition 1(f). Define $x_f{}^0 > 0$ by $f'(x_f{}^0) \equiv k^0g'(0)$ as in Proposition 1(a). At sites of type A we have $L_{Ag} = 0$ when $0 \leq L_A \leq x_f{}^0$ and $L_{Ag} > 0$ when $L_A > x_f{}^0$.

Denote the SRE population at type-A sites by $L_A(N)$, which is continuous and increasing with $L_A(0) = 0$ and $L_A(\infty) = \infty$. These properties follow from Proposition 2 and the properties of the inverse function η defined in (3). Denote the SRE population at type-B sites by $L_B(N)$.

The properties of $L_A(N)$ imply that there is a unique $N_h > 0$ such that $L_h \equiv L_A(N_h)$. Furthermore, $N \leq N_h$ implies $L_A(N) \leq L_h$ and $L_B(N) = 0$, while $N > N_h$ implies $L_A(N) > L_h$ and $L_B(N) > 0$. Similarly, there is a unique $N_f > 0$ such that $x_f{}^0 \equiv L_A(N_f)$. Furthermore, $N \leq N_f$ implies $L_A(N) \leq x_f{}^0$ and $L_{Ag}(N) = 0$, while $N > N_f$ implies $L_A(N) > x_f{}^0$ and $L_{Ag}(N) > 0$.

Assume $k^0 < \underline{k}$ as in Proposition 7(a). We want to show that this implies $N_h < N_f$. Suppose instead $N_h \geq N_f$. This implies $L_B(N_f) = 0$ and $L_A(N_f) = N_f/\lambda$. By the construction of N_h we have $\theta_A{}^*h[L_A(N_h), k^0] = \theta_B{}^*f'(0)$. By $N_f \leq N_h$, the fact that $L_A(N)$ is increasing, the fact that h is decreasing, and the earlier result $L_A(N_f) = N_f/\lambda$, we have $\theta_A{}^*h(N_f/\lambda, k^0) \geq \theta_B{}^*f'(0)$. Because gathering is not used at N_f, this reduces to $\theta_A{}^*f(N_f/\lambda)/(N_f/\lambda) \geq \theta_B{}^*f'(0)$. From (10) we have $\theta_A{}^*f(\underline{L}_A)/\underline{L}_A \equiv \theta_B{}^*f'(0)$ and together these imply $N_f/\lambda \leq \underline{L}_A$. However, by the construction of N_f, the definition of $\underline{k}$ in (10), and $k^0 < \underline{k}$ we have $k^0 = f'(N_f/\lambda)/g'(0) < f'(\underline{L}_A)/g'(0) = \underline{k}$. This implies $N_f/\lambda > \underline{L}_A$, which contradicts the previous result $N_f/\lambda \leq \underline{L}_A$. Therefore $k^0 < \underline{k}$ implies $N_h < N_f$.

Now assume $k^0 > \underline{k}$ as in Proposition 7(b). We want to show that this implies $N_h > N_f$. Suppose instead $N_h \leq N_f$. By construction we have $L_B(N_h) = 0$ and $L_A(N_h) = N_h/\lambda$. Also by the construction of N_h we have $\theta_A{}^*h(N_h/\lambda, k^0) = \theta_B{}^*f'(0)$. Because gathering is not used at N_h due to $N_h \leq N_f$, this reduces to $\theta_A{}^*f(N_h/\lambda)/(N_h/\lambda) = \theta_B{}^*f'(0)$. From (10) we have $\theta_A{}^*f(\underline{L}_A)/\underline{L}_A \equiv \theta_B{}^*f'(0)$ and thus $N_h/\lambda = \underline{L}_A$. However, by the construction of N_f, the definition of $\underline{k}$ in (10), and $k^0 > \underline{k}$ we have $k^0 = f'[L_A(N_f)]/g'(0) > f'(\underline{L}_A)/g'(0) = \underline{k}$. This implies that $L_A(N_f) < \underline{L}_A = N_h/\lambda = L_A(N_h)$. This is a contradiction because $N_h \leq N_f$ and $L_A(N)$ is increasing. Therefore $k^0 > \underline{k}$ implies $N_h > N_f$.

Finally, assume $k^0 = \underline{k}$ as in Proposition 7(c). We want to show that this implies $N_h = N_f$. Suppose instead $N_h > N_f$. Using this strict inequality in the argument from two paragraphs above, we can show that $N_f/\lambda < \underline{L}_A$. Modifying the rest of the argument using $k^0 = \underline{k}$, we can show that $N_f/\lambda = \underline{L}_A$. This is a contradiction. Next suppose instead $N_h < N_f$. Using the argument from one paragraph above, we can show that $L_A(N_h) = N_h/\lambda = \underline{L}_A$. Modifying the rest of the argument using $k^0 = \underline{k}$ gives $L_A(N_f) = \underline{L}_A = L_A(N_h)$. This is a contradiction because $N_h < N_f$ and $L_A(N)$ is an increasing function. Therefore $k^0 = \underline{k}$ implies $N_h = N_f$.

(a) Consider Proposition 7(a). Assume $k^0 < \underline{k}$ and thus $N_h < N_f$. Let N^* be the LRE population for the new climate and the productivity k^0. There are three cases:

(a)(i) Suppose $N^* \leq N_h$. This implies $L_B{}^* = 0$ in LRE. $L_B{}^* = 0$ holds iff $\theta_B{}^*h(0, k^0) \leq y^*$. Because $h(0, k^0) \equiv \max\{f'(0), k^0g'(0)\} = f'(0)$, this reduces to $\theta_B{}^*f'(0) \leq y^*$.

(a)(ii) Suppose $N_h < N^* \leq N_f$. The fact that $N_h < N^*$ implies $L_B^* > 0$ in LRE. As above, this holds iff $\theta_B^* f'(0) > y^*$. $N^* \leq N_f$ implies $L_{Ag}^* = 0$, which holds iff $f'(L_A^*) \geq k^0 g'(0)$, where L_A^* is defined by the LRE condition $\theta_A^* h(L_A^*, k^0) \equiv y^*$ stated in Proposition 7.

(a)(iii) Suppose $N_f < N^*$. The fact that $N_h < N^*$ implies $L_B^* > 0$ in LRE. As above, this holds iff $\theta_B^* f'(0) > y^*$. $N_f < N^*$ implies $L_{Ag}^* > 0$, which holds iff $f'(L_A^*) < k^0 g'(0)$, where L_A^* is defined as in case (ii) above.

These cases are mutually exclusive and exhaustive, so the converses also hold:

(a)(i) If $\theta_B^* f'(0) \leq y^*$ then $N^* \leq N_h < N_f$;

(a)(ii) If $\theta_B^* f'(0) > y^*$ and $f'(L_A^*) \geq k^0 g'(0)$ then $N_h < N^* \leq N_f$; and

(a)(iii) If $\theta_B^* f'(0) > y^*$ and $f'(L_A^*) < k^0 g'(0)$ then $N_h < N_f < N^*$.

The results in Proposition 7(a) are obtained as follows.

(a)(i) If $\theta_B^* f'(0) \leq y^*$ then $N^* \leq N_h < N_f$. We have $N^0 < N^*$ because climate amelioration implies that the baseline VLRE in Proposition 5 has a lower regional population than the new LRE. From A4 the regional population $\{N^t\}$ is increasing and $\{N^t\}$ approaches N^* in the limit. Thus $N^t < N_h < N_f$ for all $t \geq 0$. It follows that type-B sites never become active and gathering is never used. The sedentism rate remains at zero because $L_B^t = 0$ for all $t \geq 0$.

(a)(ii) If $\theta_B^* f'(0) > y^*$ and $f'(L_A^*) \geq k^0 g'(0)$ then $N_h < N^* \leq N_f$. It must be true that $N^0 \leq N_h$ because type-B sites are not active in period $t = 0$ (by assumption the necessary condition for this to occur in Proposition 6(b) does not hold). Again the regional population $\{N^t\}$ is increasing and $\{N^t\}$ approaches N^* in the limit. Thus $N^t < N_f$ for all $t \geq 0$ and gathering is never used. However, there is some $T > 0$ such that

$N^t \le N_h$ for t = 0, 1 . . T-1 and $N^t > N_h$ for t = T, T+1 . . Therefore the type-B sites are not active for t < T but are active for t ≥ T. The sedentism rate has a positive limit $S^* = L_B^*/L_A^* < 1$ because $L_A(N)$ and $L_B(N)$ are continuous, N approaches N*, and $0 < L_B^* < L_A^*$.

(a)(iii) If $\theta_B{}^*f'(0) > y^*$ and $f'(L_A^*) < k^0g'(0)$ then $N_h < N_f < N^*$. Again $N^0 \le N_h$, the regional population $\{N^t\}$ is increasing, and $\{N^t\}$ approaches N* in the limit. As above there is some T > 0 such that $N^t \le N_h$ for t = 0, 1 . . T-1 and $N^t > N_h$ for t = T, T+1 . . Therefore the type-B sites are not active for t < T but are active for t ≥ T. In addition, there is some T′ ≥ T such that $N^t \le N_f$ for t = 0, 1 . . T′-1 and $N^t > N_f$ for t = T′, T′+1 . . Therefore gathering is not used for t < T′ but is used at sites of type A for t ≥ T′. The result for S* is obtained as in case (ii) above.

(b) Consider Proposition 7(b). Assume $k^0 > \underline{k}$ and thus $N_h > N_f$. Let N* be the LRE population for the new climate and the productivity k^0. There are three cases:

(b)(i) Suppose $N^* \le N_f$. This implies $L_{Ag}^* = 0$ in LRE, which holds iff $f'(L_A^*) \ge k^0g'(0)$.

(b)(ii) Suppose $N_f < N^* \le N_h$. This implies $L_{Ag}^* > 0$ in LRE, which holds iff $f'(L_A^*) < k^0g'(0)$. $N^* \le N_h$ implies $L_B^* = 0$ in LRE, which holds iff $\theta_B{}^*h(0, k^0) \le y^*$. From $h(0, k^0) = f'(0)$ this reduces to $\theta_B{}^*f'(0) \le y^*$.

(b)(iii) Suppose $N_h < N^*$. This implies $f'(L_A^*) < k^0g'(0)$ as above. $N_h < N^*$ implies $L_B^* > 0$ in LRE, which holds iff $\theta_B{}^*f'(0) > y^*$.

These cases are mutually exclusive and exhaustive, so the converses also hold:

(b)(i) If $f'(L_A^*) \ge k^0g'(0)$ then $N^* \le N_f < N_h$;

(b)(ii) If $f'(L_A^*) < k^0g'(0)$ and $\theta_B{}^*f'(0) \le y^*$ then $N_f < N^* \le N_h$; and

(b)(iii) If $f'(L_A^*) < k^0g'(0)$ and $\theta_B{}^*f'(0) > y^*$ then $N_f < N_h < N^*$.

The results in Proposition 7(b) are obtained as follows.

(b)(i) If $f'(L_A^*) \geq k^0 g'(0)$ then $N^* \leq N_f < N_h$. We have $N^0 < N^*$ because climate amelioration implies that the baseline VLRE in Proposition 5 has a lower regional population than the new LRE. From A4, regional population $\{N^t\}$ is increasing and $\{N^t\}$ approaches N^* in the limit. Thus $N^t < N_f < N_h$ for all $t \geq 0$. It follows that type-B sites are never active and gathering is never used. The sedentism rate remains at zero because $L_B^t = 0$ for all $t \geq 0$.

(b)(ii) If $f'(L_A^*) < k^0 g'(0)$ and $\theta_B^* f'(0) \leq y^*$ then $N_f < N^* \leq N_h$. It must be true that $N^0 \leq N_f$ because gathering is not used in period $t = 0$ due to Proposition 6(a). Again the regional population $\{N^t\}$ is increasing and $\{N^t\}$ approaches N^* in the limit. Thus $N^t < N_h$ for all $t \geq 0$ and type-B sites are never active. However, there is some $T > 0$ such that $N^t \leq N_f$ for $t = 0, 1 \,..\, T\text{-}1$ and $N^t > N_f$ for $t = T, T+1 \,..$ Thus gathering is not used for $t < T$ but it is used at sites of type A for $t \geq T$. The sedentism rate remains at zero because $L_B^t = 0$ for all $t \geq 0$.

(b)(iii) If $f'(L_A^*) < k^0 g'(0)$ and $\theta_B^* f'(0) > y^*$ then $N_f < N_h < N^*$. Again $N^0 \leq N_f$, the regional population $\{N^t\}$ is increasing, and $\{N^t\}$ approaches N^* in the limit. As above there is some $T > 0$ such that $N^t \leq N_f$ for $t = 0, 1 \,..\, T\text{-}1$ and $N^t > N_f$ for $t = T, T+1 \,..$ Thus gathering is not used for $t < T$ but it is used at sites of type A for $t \geq T$. In addition, there is some $T' \geq T$ such that $N^t \leq N_h$ for $t = 0, 1 \,..\, T'\text{-}1$ and $N^t > N_h$ for $t = T', T'+1 \,..$ Thus type-B sites are not active for $t < T'$ but are active for $t \geq T'$. The sedentism rate has a positive limit $S^* = L_B^*/L_A^* < 1$ because $L_A(N)$ and $L_B(N)$ are continuous, N approaches N^*, and $0 < L_B^* < L_A^*$.

(c) Consider Proposition 7(c). Assume $k^0 = \underline{k}$ and thus $N_h = N_f$. Let N* be the LRE population for the new climate and the productivity k^0. There are two cases:

(c)(i) Suppose $N^* \leq N_f = N_h$. This implies $L_B^* = 0$ in LRE, which holds iff $\theta_B^* f'(0) \leq y^*$. It also implies that gathering does not occur at type-A sites in LRE, which is true iff $f'(L_A^*) \geq k^0 g'(0)$.

(c)(ii) Suppose $N_f = N_h < N^*$. This implies $L_B^* > 0$ in LRE, which holds iff $\theta_B^* f'(0) > y^*$. It also implies that gathering does occur at type-A sites in LRE, which is true iff $f'(L_A^*) < k^0 g'(0)$.

These cases are mutually exclusive and exhaustive, so the converses also hold:

(c)(i) If $\theta_B^* f'(0) \leq y^*$ and $f'(L_A^*) \geq k^0 g'(0)$ then $N^* \leq N_f = N_h$.

(c)(ii) If $\theta_B^* f'(0) > y^*$ and $f'(L_A^*) < k^0 g'(0)$ then $N_f = N_h < N^*$.

The results in Proposition 7(c) are obtained as follows.

(c)(i) If $\theta_B^* f'(0) \leq y^*$ and $f'(L_A^*) \geq k^0 g'(0)$ then $N^* \leq N_f = N_h$. We have $N^0 < N^*$ due to climate amelioration. As in other cases, the regional population $\{N^t\}$ is increasing and approaches N* in the limit. Thus $N^t < N_f = N_h$ for all $t \geq 0$. It follows that type-B sites are never active and gathering is never used. The sedentism rate remains at zero because $L_B^t = 0$ for all $t \geq 0$.

(c)(ii) If $\theta_B^* f'(0) > y^*$ and $f'(L_A^*) < k^0 g'(0)$ then $N_f = N_h < N^*$. It must be true that $N^0 \leq N_f = N_h$ because gathering is not used in period t = 0 due to Proposition 6(a). The path $\{N^t\}$ has the same properties as in case (i) above. Thus there is some T > 0 such that $N^t \leq N_f = N_h$ for t = 0, 1 . . T-1 and $N^t > N_f = N_h$ for t = T, T+1 . . It follows that type-B sites are not active and gathering is not used for t < T, but type-B sites become active and gathering is used at type-A sites for $t \geq T$. The sedentism rate has a positive limit $S^* = L_B^*/L_A^* < 1$ because $L_A(N)$ and $L_B(N)$ are continuous, N approaches N*, and $0 < L_B^* < L_A^*$.

Proof of Proposition 8.

Because gathering never shuts down after it begins, the only possibility for VLRE involves gathering productivity at the level k*. The conditions for LRE require that sites of type B have $L_B^* = 0$ if $\theta_B^*h(0, k^*) \leq y^*$ and $L_B^* > 0$ if $\theta_B^*h(0, k^*) > y^*$.

In cases (a)(iii), (b)(iii), and c(ii) from Proposition 7, we have $\theta_B^*f'(0) > y^*$. By the definition in Proposition 1(f), $h(0, k^*) \equiv \max \{f'(0), k^*g'(0)\} \geq f'(0)$. Therefore in all of these cases we have $\theta_B^*h(0, k^*) > y^*$. This implies $L_B^* > 0$ in the new VLRE.

In case (b)(ii) from Proposition 7, we have $\theta_B^*f'(0) \leq y^*$. From the definition of $h(0, k^*)$, the inequality $\theta_B^*h(0, k^*) > y^*$ holds iff $\theta_B^*k^*g'(0) > y^*$. This implies $L_B^* > 0$ iff the latter inequality holds.

Proof of Proposition 9.

The baseline VLRE in Proposition 5 has $\theta_A^0f'(0) > y^*$ due to (9a). Suppose that only gathering is used at the type-A sites in the new VLRE. This implies $f'(0) \leq kg'(L_A^*)$ where L_A^* is the local population at type-A sites and k is the gathering productivity in the new VLRE. It is unimportant whether $k < k^*$ or $k = k^*$. If only gathering is used at the type-A sites, then LRE implies $\theta_A^*kg(L_A^*)/L_A^* = y^*$. Combining these results with the fact that the average product of gathering exceeds the marginal product, we obtain

$$y^* < \theta_A^0f'(0) < \theta_A^*f'(0) \leq \theta_A^*kg'(L_A^*) < \theta_A^* kg(L_A^*)/L_A^* = y^*$$

This is a contradiction. It follows that type-A sites must use both hunting and gathering in the new VLRE.